Local Type Inference with Symbolic Closures

Ambrose Bonnaire-Sergeant

What is Local Type Inference?

Partially-annotated programs → *Local type inference* → System F

1. Bidirectional type checking
2. Parameter type inference
3. Type argument inference

Bidirectional checking

▲ Synthesis mode (types propagate up)

▼ Checking mode (types propagate down)

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$$\frac{}{\Gamma \vdash \texttt{n} \blacktriangle \texttt{Int}} \qquad \frac{}{\Gamma \vdash \texttt{s} \blacktriangle \texttt{Str}}$$

$$\frac{\texttt{e} \blacktriangledown \texttt{Int}}{\Gamma \vdash \texttt{(inc e)} \blacktriangle \texttt{Int}}$$

Bidirectional checking

▲ Synthesis mode (types propagate up)

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$$\frac{\Gamma \vdash \texttt{e} \blacktriangle \texttt{T}}{\Gamma \vdash \texttt{e} \blacktriangledown \texttt{T}}$$

$$\frac{\texttt{e} \blacktriangledown \texttt{Int}}{\Gamma \vdash \texttt{(inc e)} \blacktriangle \texttt{Int}}$$

$$\frac{\Gamma,\ \texttt{x:T} \vdash \texttt{e} \blacktriangle \texttt{S}}{\Gamma \vdash \texttt{(}\lambda\ \texttt{(x : T) e)} \blacktriangledown \texttt{T -> S}}$$

Bidirectional checking

$\blacktriangle$ Synthesis mode (types propagate up)

$\blacktriangledown$ Checking mode (types propagate down)

$$\dfrac{\dfrac{\Gamma \vdash \texttt{1} \blacktriangle \texttt{Int}}{\Gamma \vdash \texttt{1} \blacktriangledown \texttt{Int}}}{\Gamma \vdash \texttt{(inc 1)} \blacktriangle \texttt{Int}}$$

Example: Checking `(inc 1)`

Bidirectional checking

$\blacktriangle$ Synthesis mode (types propagate up)

$\blacktriangledown$ Checking mode (types propagate down)

Simple for implementors and users to conceptualize ✓

Yields predictable, local error messages ✓

$$\frac{\dfrac{\Gamma \vdash \texttt{1} \blacktriangle \texttt{Int}}{\Gamma \vdash \texttt{1} \blacktriangledown \texttt{Int}}}{\Gamma \vdash \texttt{(inc 1)} \blacktriangle \texttt{Int}}$$

Example: Checking `(inc 1)`

Parameter type inference

Input (Clojure)

```
(ann (fn [x] (inc x)) [Int -> Int])
```

Infer function parameter types

Output (System F)

```
(fn [x :- Int] (inc x))
```

Type Argument Reconstruction

Input (Clojure)

```
(map inc [1 2 3])
```

Infer type arguments

Output (System F)

```
(map<Int,Int> inc [1 2 3])
```

The “Hard-to-Synthesize Arguments” Problem

```
(map (fn [x] (inc x)) [1 2 3])
```

The "Hard-to-Synthesize Arguments" Problem

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(map (fn [x] (inc x)) [1 2 3])
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Cannot simultaneously infer type arguments to `map` and missing parameter type

✘

The “Hard-to-Synthesize Arguments” Problem

```
(map (fn [x] (inc x)) [1 2 3])
```

Cannot simultaneously infer type arguments to `map` and missing parameter type

Why?

To infer type arguments,
you must first synthesize types for operands…

…but unannotated functions are hard-to-synthesize types for!

Existing solutions

Typed {Racket,Clojure} `Note: Any = ⊤`

Still doesn't check! ✗

```
(map (fn [x :- Any] (inc x))
     [1 2 3])
```

TypeScript `Note: any ≈ (void*)`

Function body is trusted! ✗

```
[1,2,3].map((x:any)=>x+1)
```

Reticulated Python

Runtime overhead ✗

```
map(lambda (x:Dyn): x+1,
    [1,2,3])
```

Existing solutions

Java Lambdas

```
List.of(1,2,3)
.map(x->x+1)
```

Type args

Param type (inferred as Int)

Gold standard

Java Lambdas

```
roster
    .stream()
    .filter(
        p -> p.getGender() == Person.Sex.MALE
            && p.getAge() >= 18
            && p.getAge() <= 25)
    .map(p -> p.getEmailAddress())
    .forEach(email -> System.out.println(email));
```

Type args
Param type
Type args
Param type
Type args
Param type

...is this achievable with non-OO idioms?

Solving the
"Hard-to-synthesize arguments"
problem with Symbolic Analysis

Another hard-to-synthesize term

```
(let [f (fn [x] x)]
  (f 1)
  (f ‘‘a’’))
```

How to check?

Wishful thinking

```
(let [f (fn [x] x)]
  (f 1)
  (f “a”))
```

1. Infer polymorphic principal(-like) type for f

```
(let [f (ann (fn [x] x)
             (All [a] [a -> a]))]
  (f 1)
  (f “a”))
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1. Infer polymorphic principal(-like) type for f

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(let [f (ann (fn [x] x)
             (All [a] [a -> a]))]
  (f 1)
  (f ‘‘a’’))
```

2. Infer sufficiently capable intersection type for f

```
(let [f (ann (fn [x] x)
             (IFn [Int -> Int]
                  [Str -> Str]))]
  (f 1)
  (f ‘‘a’’))
```

Wishful thinking

```
(let [f (fn [x] x)]
  (f 1)
  (f ‘‘a’’))
```

<u>This talk:</u>
Achieving this transformation within the framework of Local Type Inference

1. Infer polymorphic principal(-like) type for f

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Challenges

```
(let [f (fn [x] x)]
  (f 1)
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Posed by Hosoya & Pierce,
“How Good is Local Type Inference?” (1999)

Challenges

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(let [f (fn [x] x)]
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1. How to delay the checking of hard-to-synthesize terms?

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1. How to delay the checking of hard-to-synthesize terms?

2. How to force checking of hard-to-synthesize terms to preserve soundness?

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Idea 1: Inline let-bound functions

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  (f 1)
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↓

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(let []
  ((fn [x] x) 1)
  ((fn [x] x) ‘‘a’’))
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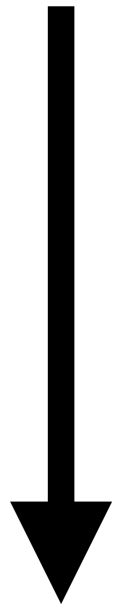

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   A: Inline let-bound unannotated functions

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   A: Automatic

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(let []
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```

✗ does not terminate if f is recursive

✗ how to determine if a variable binds an (unannotated) function?

1. How to delay the checking of hard-to-synthesize terms?

   A: Inline let-bound unannotated functions

2. How to force checking of hard-to-synthesize terms to preserve soundness?

   A: Automatic

✗ ***Problem: Variable-capture***

Idea 1: Inline let-bound functions

```
(let [f (let [y <DB-write>]
          (fn [x] y y))]
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?

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(let []
  ((let [y <DB-write>]
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```

Idea 1: Inline let-bound functions

```
(let [f (let [y <DB-write>]
          (fn [x] y y))]
  (f 1)
  (f "a"))
```

? ?

```
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  ((let [y <DB-write>]
     (fn [x] y y))
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  ((let [y <DB-write>]
     (fn [x] y y))
   "a"))
```

```
(let []
  ((fn [x] <DB-write> <DB-write>)
   1)
  ((fn [x] <DB-write> <DB-write>)
   "a"))
```

Idea 2: Let-polymorphism

```
(let [f (fn [x] x)]
  (f 1)
  (f ‘‘a’’))
```

Let-polymorphism infers a principal type scheme for `f` and copies the type (with renamed unification variables) in each occurrence of `f` for separate instantiation.

...immediately doesn’t work because f’s type is hard-to-synthesize!
(no unification variables in Local Type Inference)

Idea 3: “Delayed function type”

```
(let [f (fn [x] x)]
  (f 1)
  (f ‘‘a’’))
```

Idea 3: "Delayed function type"

```
(let [f (fn [x] x)]
  (f 1)
  (f ''a''))
```

```
f : @(fn [x] x)
```

1. How to delay the checking of hard-to-synthesize terms?

A: Introduction rule for unannotated functions makes a "delayed function type"

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   A: Applications of delayed function types rechecks the function's source code with given argument types

```
@(fn [x] x) <: Int -> ?
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```
@(fn [x] x) <: Int -> ?
@(fn [x] x) <: Str -> ?
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Idea 3: “Delayed function type”

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   A: Applications of delayed function types rechecks the function’s source code with given argument types

```
@(fn [x] x) <: Int -> ?
@(fn [x] x) <: Str -> ?
```

<u>Problem: Undecidable!</u>

Idea 3: “Delayed function type”

```
(let [f (fn [f] (f f))]
  (f f))
```

1. Delay `(fn [f] (f f))`
2. Check `(f f)`
3. Check `(f f)`
4. Check `(f f)`

…

Problem: Undecidable!

Restrictions

Insight:
Many local functions are not recursive
(implicitly or explicitly)

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Most top-level functions have annotations
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New Restrictions:

1. Only delay local functions
2. Do not allow delayed functions to escape its top-level form
3. Use fuel to make uncommon cases (recursive locals) conservatively decidable

Idea 3: “Delayed function type”

```
(let [f (fn [f] (f f))]
  (f f))
```

1. Delay `(fn [f] (f f))`
2. Check `(f f) Fuel = 2`
3. Check `(f f) Fuel = 1`
4. Check `(f f) Fuel = 0`
5. Type error: Reduction limit

Tradeoff: Platform dependency

Idea 3: “Delayed function type”

```
(let [f (let [y 1]
          (fn [x] y))]
  (f 1)
  (f ‘‘a’’))
```

Problem: Variable Capture!

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Idea 3: "Delayed function type"

```
(let [f (let [y 1]
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  (f 1)
  (f ''a''))
```

Lost the type of y!

```
f : @(fn [x] y)
```

<u>Problem: Variable Capture!</u>

Solution: Symbolic *Closures*

```
(let [f (let [y 1]
          (fn [x] y))]
  (f 1)
  (f ‘‘a’’))
```

```
f : y:Int@(fn [x] y)
```

Keep ***type*** environment for when we need it (“type-level” closure)

Solution: Symbolic *Closures*

```
(let [f (let [y 1]
             (fn [x] y))]
  (f 1)
  (f ‘‘a’’))
```

Can check y!

```
f : y:Int@(fn [x] y)
```

Keep ***type*** environment for when we need it (“type-level” closure)

Example Elaboration

Elaboration with Symbolic Closures

Input

```
(let [f (fn [x] x)]
  (f 1)
  (f ‘‘a’’))
```

Output

```
(let [f (ann (fn [x] x)
             (IFn [Int -> Int]
                  [Str -> Str]))]
  (f 1)
  (f ‘‘a’’))
```

Elaboration with Symbolic Closures

Input

```
(let [f (fn [x] x)]
  (f 1)
  (f "a"))
```

→ 1. Assign f a symbolic closure: `f : {}@(fn [x] x)`

Output

```
(let [f (ann (fn [x] x)
             (IFn [Int -> Int]
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  (f "a"))
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Elaboration with Symbolic Closures

Input

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(let [f (fn [x] x)]
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1. Assign f a symbolic closure: `f : {}@(fn [x] x)`

2. Check `f` with Int (returns Int) `f <: Int -> ?`

Output

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(let [f (ann (fn [x] x)
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Elaboration with Symbolic Closures

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Elaboration with Symbolic Closures

Input

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(let [f (fn [x] x)]
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   (f ‘‘a’’))
```

1. Assign f a symbolic closure: `f : {}@(fn [x] x)`
2. Check `f` with Int (returns Int) `f <: Int -> ?`
3. Check `f` with Str (returns Str) `f <: Str -> ?`
4. Replace f’s type with its capabilities

Output

```
(let [f (ann (fn [x] x)
             (IFn [Int -> Int]
                  [Str -> Str]))]
   (f 1)
   (f ‘‘a’’))
```

End example,
break for questions?

More about Symbolic Closures

C-AppClosure

$$\frac{\Gamma \vdash f : \Gamma' @ \lambda x.e' \qquad \Gamma \vdash^{c} e : \sigma \qquad \Gamma', x : \sigma \vdash e' : \tau}{\Gamma \vdash^{c} (f\ e) : \tau}$$

C-AppClosure

$$\frac{\Gamma \vdash f : \Gamma'@\lambda x.e' \qquad \Gamma \vdash^{c} e : \sigma \qquad \Gamma', x : \sigma \vdash e' : \tau}{\Gamma \vdash^{c} (f\ e) : \tau}$$

SC-Closure

$$\frac{\Gamma, \overline{\alpha}, x : \tau \vdash e : \sigma}{\Gamma@\lambda x.e \leq (\tau \xrightarrow{\overline{\alpha}} \sigma)}$$

$$\textsc{C-AppClosure}\quad \frac{\Gamma \vdash f : \Gamma'@\lambda x.e' \qquad \Gamma \vdash^{c} e : \sigma \qquad \Gamma', x : \sigma \vdash e' : \tau}{\Gamma \vdash^{c} (f\ e) : \tau}$$

Subtyping relation calls type checker

$$\textsc{SC-Closure}\quad \frac{\Gamma, \overline{\alpha}, x : \tau \vdash e : \sigma}{\Gamma@\lambda x.e \leq (\tau \xrightarrow{\overline{\alpha}} \sigma)}$$

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Via subsumption rule

How to check?

```
(map (fn [x] (inc x))
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Derive data-flow graph from operator

```
(All [a b]
  [[a -> b] (Seqable a) -> (Seq b)])
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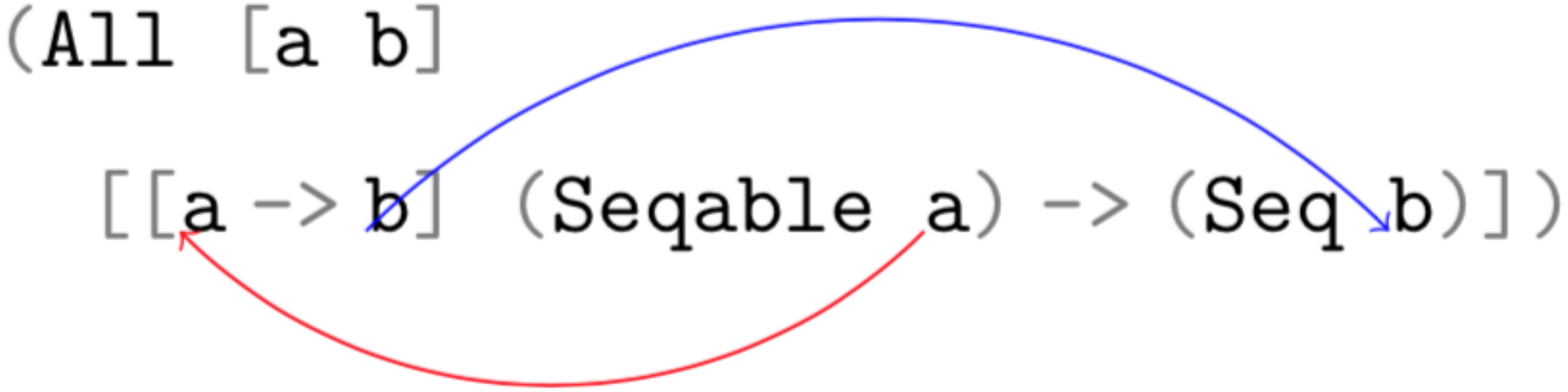
Derive data-flow graph from operator



Solve constraints to a fixed point

```
{}@(fn [x] (inc x)) <: [a -> b]       => C1
(Vec Number)        <: (Seqable a) => C2
```

How to check?

```
(map (fn [x] (inc x))
     [1 2 3])
```

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(All [a b]
  [[a -> b] (Seqable a) -> (Seq b)])
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Solve constraints to a fixed point

```
{}@(fn [x] (inc x)) <: [a -> b]       => C1
(Vec Number)        <: (Seqable a) => C2
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Future work: What if data-flow is recursive?

Related work

Related work

Expansion variables

$$\langle (z : a) \vdash (((a \to b) \to b) \to c) \to c \rangle$$

$$\Downarrow$$

$$\langle (z : a_1 \sqcap a_2) \vdash (((a_1 \to b_1) \to b_1) \sqcap ((a_2 \to b_2) \to b_2) \to c) \to c \rangle$$

Similar goal as
"Expansion variables" in
Intersection Type Inference

Similar cost:
Inference cost = Beta-reduction cost

Carlier & Wells' System E (2004)

Related work

Colored Local Type Inference

Allows partial type information to propagate down term

For instance, if **g** is known to have type $\forall \mathsf{a}.(\mathsf{Int} \rightarrow \mathsf{a}) \rightarrow \mathsf{a}$, then

$$\mathbf{g}\ (\mathbf{fun}\ (\mathsf{x})\ \mathsf{x} + 1)$$

Conservative extension of Local Type Inference

Odersky et al. Colored Local Type Inference (POPL 2001)

Review

Background:
Local type inference requires annotations

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Problem:
Local annotations are annoying

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Insight:
Top-level annotations are provided

Insight:
Local functions are usually trivial

Review

Background:
Local type inference requires annotations

Problem:
Local annotations are annoying

Insight:
Top-level annotations are provided

Solution:
Use symbolic analysis
to infer simple local functions

Insight:
Local functions are usually trivial

Thanks!